Homework 5 – Answers to Questions

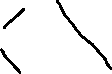
Jade Pearl

CMSC 451-7380

Professor Dennis Didulo

**Question 1**:

|  |  |  |
| --- | --- | --- |
| A | U |  |
| B | U |  |
| C | U |  |
| D | U |  |
| E | U |  |
| F | F | 0 |
| G | U |  |
| H | U |  |
| I | U |  |



minimum



|  |  |  |
| --- | --- | --- |
| A | F | 7 |
| B | U |  |
| C | U |  |
| D | U |  |
| E | F | 6 |
| F | T |  |
| G | U |  |
| H | U |  |
| I | F | 5 |



minimum

|  |  |  |
| --- | --- | --- |
| A | F | 7 |
| B | U |  |
| C | U |  |
| D | U |  |
| E | F | 2 |
| F | T |  |
| G | F | 4 |
| H | F | 6 |
| I | T |  |



Min updated



|  |  |  |
| --- | --- | --- |
| A | F | 7 |
| B | F | 1 |
| C | U |  |
| D | U |  |
| E | T |  |
| F | T |  |
| G | F | 4 |
| H | F | 6 |
| I | T |  |



Min updated



|  |  |  |
| --- | --- | --- |
| A | F | 7 |
| B | T |  |
| C | F | 5 |
| D | U |  |
| E | T |  |
| F | T |  |
| G | F | 4 |
| H | F | 3 |
| I | T |  |



min updated



|  |  |  |
| --- | --- | --- |
| A | F | 7 |
| B | T |  |
| C | F | 2 |
| D | U |  |
| E | T |  |
| F | T |  |
| G | F | 4 |
| H | T |  |
| I | T |  |



Min updated



|  |  |  |
| --- | --- | --- |
| A | F | 7 |
| B | T |  |
| C | T |  |
| D | F | 1 |
| E | T |  |
| F | T |  |
| G | F | 4 |
| H | T |  |
| I | T |  |



Min updated



|  |  |  |
| --- | --- | --- |
| A | F | 2 |
| B | T |  |
| C | T |  |
| D | T |  |
| E | T |  |
| F | T |  |
| G | F | 4 |
| H | T |  |
| I | T |  |

min updated



|  |  |  |
| --- | --- | --- |
| A | T |  |
| B | T |  |
| C | T |  |
| D | T |  |
| E | T |  |
| F | T |  |
| G | F | 3 |
| H | T |  |
| I | T |  |



minimum



|  |  |  |
| --- | --- | --- |
| A | T |  |
| B | T |  |
| C | T |  |
| D | T |  |
| E | T |  |
| F | T |  |
| G | T |  |
| H | T |  |
| I | T |  |



minimum (total weight 19)



**Question 2**:

Below is the execution of Kruskal’s algorithm on the given graph:

Choose DM 1 (D, M)  
Choose GH 1 (D, M) (G, H)  
Choose JM 1 (D, J, M) (G, H)  
Choose EJ 2 (D, E, J, M) (G, H)  
Choose HM 2 (D, E, G, H, J, M)  
Choose DF 3 (D, E, F, G, H, J, M)  
Skip FM 3  
Choose KL 3 (D, E, F, G, H, J, M) (K, L)  
Choose AD 4 (A, D, E, F, G, H, J, M) (K, L)  
Choose BC 4 (A, D, E, F, G, H, J, M) (B, C) (K, L)  
Choose CM 5 (A, B, C, D, E, F, G, H, J, M) (K, L)  
Skip EF 5, Skip GM 5  
Choose JK 5 (A, D, E, F, G, H, J, K, L, M)  
Skip DE 6  
Choose IM 7 (A, D, E, F, G, H, I, J, K, L, M)

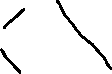
Here is the minimum spanning tree produced by Kruskal’s algorithm (total weight 38):



**Question 3**:

To determine if an MST is unique, you must consider the edge weights of an undirected, weighted graph, *G*. According to Lemma 7.1 as found by the University of Illinois, if all edge weights in a connected graph *G* are distinct, then *G* has a unique minimum spanning tree.

Here is the graph and MST from problem 1:



Total weight: 19



The graph does NOT have distinct weights, therefore, it may not have a unique MST. Since this tree is small, we can go by trial and error to see if we can find a different MST that has the same total weight as the MST found with Prim’s. We can use Kruskal’s algorithm as well. In this case. Kruskal’s algorithm yields the same results as the MST found with Prim’s algorithm. Through trial and error, I could not find any alternative MSTs that have a total weight of 19 as found by Prim’s and Kruskal’s algorithms. Therefore this graph has a unique MST.

The graph from the second problem does have an alternative MST than the one found by Kruskal’s with the same weight:



**Question 4**:



|  |  |  |
| --- | --- | --- |
| A | T |  |
| B | F | 2 minimum |
| C | F | 7 |
| D | F | 6 |
| E | U |  |
| F | U |  |



|  |  |  |
| --- | --- | --- |
| A | T |  |
| B | T |  |
| C | F | 5 |
| D | F | 6 |
| E | U |  |
| F | F | 3 minimum |



|  |  |  |
| --- | --- | --- |
| A | T |  |
| B | T |  |
| C | F | 5 |
| D | F | 4 minimum |
| E | U |  |
| F | T |  |

|  |  |  |
| --- | --- | --- |
| A | T |  |
| B | T |  |
| C | F | 5 minimum |
| D | T |  |
| E | F | 7 |
| F | T |  |



|  |  |  |
| --- | --- | --- |
| A | T |  |
| B | T |  |
| C | T |  |
| D | T |  |
| E | F | 7 minimum |
| F | T |  |



|  |  |  |
| --- | --- | --- |
| A | T |  |
| B | T |  |
| C | T |  |
| D | T |  |
| E | T |  |
| F | T |  |

**Resources:**

(n.d.). *Minimum Spanning Trees* (pp. 257–272) [Review of *Minimum Spanning Trees*]. University of Illinois Urbana-Champaign. Retrieved November 28, 2023, from <https://jeffe.cs.illinois.edu/teaching/algorithms/book/07-mst.pdf>